5/ Tableau des primitives usuelles

Fonction	Primitive
$\frac{1}{x}$	$\ln\lvert x\rvert$
x^r ; $r\neq -1$	$\frac{x^{r+1}}{r+1}$
e^x , e^{ax}	e^x, $\frac{e^{ax}}{a}$
$\cos x$, $\cos ax$	$\sin x$; $\frac{\sin ax}{a}$
$\sin x$; $\sin ax$	$-\cos x$; $-\frac{\cos ax}{a}$
$\frac{1}{\cos^2 x}$; $\frac{1}{\cos^2 ax}$	$\tan x$; $\frac{\tan ax}{a}$
$\frac{1}{\sin^2 x}$; $\frac{1}{\sin^2 ax}$	$-\operatorname{cotan} x$; $-\frac{\operatorname{cotan} x}{a}$
$\frac{1}{\sin x}$, $\frac{1}{\cos x}$	$\ln\lvert\tan\frac{x}{2}\rvert$; $\ln\lvert\tan(\frac{x}{2}+\frac{\pi}{4})\rvert$
$\frac{1}{\sin x\cos x}$	$\ln\lvert\tan x\rvert$
$\operatorname{ch} x$, $\operatorname{ch} ax$	$\operatorname{sh} x$, $\frac{\operatorname{sh} ax}{a}$
$\operatorname{sh} x$, $\operatorname{sh} ax$	$\operatorname{ch} x$, $\frac{\operatorname{ch} ax}{a}$
$\frac{1}{\operatorname{ch} x}$, $\frac{1}{\operatorname{sh} x}$	$2\operatorname{Arctan}(e^x)$; $\ln\lvert\operatorname{th}\frac{x}{2}\rvert$
$\frac{1}{\operatorname{ch}^2 x}$; $\frac{1}{\operatorname{ch}^2 ax}$	$\operatorname{th} x$; $\frac{\operatorname{th} ax}{a}$
$\frac{1}{\operatorname{sh}^2 x}$; $\frac{1}{\operatorname{sh}^2 ax}$	$-\frac{1}{\operatorname{th} x}$, $-\frac{1}{a\operatorname{th} ax}$
$\frac{1}{x^2+1}$; $\frac{1}{x^2+a^2}$	$\operatorname{Arctan} x$, $\frac{1}{a}\operatorname{Arctan}\frac{x}{a}$
$\frac{1}{x^2-1}$; $\frac{1}{x^2-a^2}$	$\frac{1}{2}\ln\lvert\frac{x-1}{x+1}\rvert$; $\frac{1}{2a}\ln\lvert\frac{x-a}{x+a}\rvert$
$\frac{1}{\sqrt{1-x^2}}$; $\frac{1}{\sqrt{a^2-x^2}}$	$\operatorname{Arcsin} x$; $\operatorname{Arcsin}\frac{x}{\lvert a\rvert}$
$\frac{1}{\sqrt{x^2+1}}$; $\frac{1}{\sqrt{x^2+a^2}}$	$\operatorname{Argsh} x$; $\operatorname{Argsh}\frac{x}{\lvert a\rvert}$
$\frac{1}{\sqrt{x^2-1}}$; $\frac{1}{\sqrt{x^2-a^2}}$	$\operatorname{Argch} x$; $\begin{cases}\operatorname{Argch}\frac{x}{\lvert a\rvert} & \text{si } x>\lvert a\rvert\\ \operatorname{Argch}\lvert\frac{x}{a}\rvert & \text{si } x<-\lvert a\rvert\end{cases}$

Fonction	Primitive
$\frac{u'}{u}$	$\ln\lvert u\rvert$
$u'u^r$	$\frac{u^{r+1}}{r+1}$
a^x ; $u'e^u$	$\frac{a^x}{\ln a}$, e^u
$u'\cos u$	$\sin u$
$u'\sin u$	$-\cos u$
$\frac{u'}{\cos^2 u}$	$1+\tan^2 u$
$\frac{u'}{\sin^2 u}$	$-\operatorname{cotan} u$
$\tan x$, $\frac{1}{\tan x}$	$-\ln\lvert\cos x\rvert$; $\ln\lvert\sin x\rvert$
$u'\operatorname{ch} u$	$\operatorname{sh} u$
$u'\operatorname{sh} u$	$\operatorname{ch} u$
$\operatorname{th} x$, $\frac{1}{\operatorname{th} x}$	$\ln\lvert\operatorname{ch} x\rvert$; $\ln\lvert\operatorname{sh} x\rvert$
$\frac{u'}{\operatorname{ch}^2 u}$;	$\operatorname{th} x$
$\frac{u'}{\operatorname{sh}^2 u}$	$-\frac{1}{\operatorname{th} u}$
$\frac{u'}{u^2+1}$; $\frac{u'}{u^2+a^2}$	$\operatorname{Arctan} u$, $\frac{1}{a}\operatorname{Arctan}\frac{u}{a}$
$\frac{u'}{\ldots-1}$; $\frac{u'}{u^2-\ldots}$	$\frac{1}{2}\ln\lvert\frac{u-1}{u+1}\rvert$; $\frac{1}{2a}\ln\lvert\frac{u-a}{\ldots+a}\rvert$
$\frac{u'}{\sqrt{1-u^2}}$; $\frac{u'}{\sqrt{a^2-u^2}}$	$\operatorname{Arcsin} u$; $\operatorname{Arcsin}\frac{u}{\lvert a\rvert}$
$\frac{u'}{\sqrt{u^2+1}}$; $\frac{u'}{\sqrt{u^2+a^2}}$	$\operatorname{Argsh} u$ $\operatorname{Argsh}\frac{u}{\lvert a\rvert}$
$\frac{u'}{\sqrt{u^2+b}}$	$\ln(x+\sqrt{u^2+b})$

Rappel : $\operatorname{Argsh} x = \ln(x+\sqrt{x^2+1})$; $\operatorname{Argch} x = \ln(x+\sqrt{x^2-1})$

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$$f(x) = \frac{b}{x^2+1} + \frac{d}{(x^2+1)^2} + \frac{e}{x-1} + \frac{f}{x+1}$$

$$\text{Arctan } x = \int \frac{1}{x^2+1}\,dx = \frac{x}{x+1} + 2\int \frac{x^2}{(x^2+1)^2}\,dx = \frac{x}{x^2+1} + 2\int \frac{x^2+1-1}{(x^2+1)^2}\,dx$$

$$\begin{cases} u = \frac{1}{x^2+1} \\ v' = 1 \end{cases} \qquad \begin{cases} u' = \frac{-2x}{(x^2+1)^2} \\ v = x \end{cases}$$

$$\text{Arctan } x = \int \frac{1}{x^2+1}\,dx = \frac{x}{x^2+1} + 2\left(\int \frac{1}{x^2+1}\,dx - \int \frac{1}{(x^2+1)^2}\,dx \right)$$

$$= \frac{x}{x^2+1} + 2\left(\text{Arctan } x - I \right)$$

$$2I = \frac{x}{x^2+1} + \text{Arctan } x$$

$$I = \frac{1}{2}\left(\frac{x}{x^2+1} + \text{Arctan } x \right)$$

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